Final Project

Aaron Welles, Alex Fairly, Roman

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Free Particle on a Line

Lets first consider a particle of mass m moving on a line and free of any external forces. Because the particle is free of forces there is no potential energy, and the only energy in the system is the kinetic energy. The Hamiltonian function is given as

$H = \frac{1}{2m}p^2$

$X_h = X_r \frac{\partial}{\partial r} + X_p \frac{\partial}{\partial p}$, with $X_r = \frac{dr}{dt}$ and $X_p = \frac{dp}{dt}$

$\iota_{X_H}\omega = dp \wedge dr(X_H, v) = x_p v_r - x_r v_p$

*** NOTE: flush this derivation out

$-dH = -\frac{1}{m}p\ dp = -\frac{1}{m}p\ v_p$

Since $\iota_{X_H}\omega = -dH$

$-\frac{1}{m}p\ v_p = x_p v_r - x_r v_p$,

Therefore, since v_r and v_p are linearly independent, we obtain

$X_H = \frac{1}{m}p\frac{\partial}{\partial r}$.
Therefore $\frac{1}{m}p\frac{\partial}{\partial r} = \frac{dr}{dt}\frac{\partial}{\partial r} + \frac{dp}{dt}\frac{\partial}{\partial p}$

Since $\frac{\partial}{\partial p}$ and $\frac{\partial}{\partial r}$ are linearly independent,

$\frac{d}{dt}r = \frac{1}{m}p$ and $\frac{d}{dt}p = 0$.

This means that a free particle moves with constant momentum along the line.